

Quantum-disordered slave-boson theory of underdoped cuprates

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We study the stability of the spin gap phase in the $U(1)$ slave-boson theory of the $t - J$ model in connection to the underdoped cuprates. We approach the spin gap phase from the superconducting state and consider the quantum phase transition of the slave-bosons at zero temperature by introducing vortices in the boson superfluid. At finite temperatures, the properties of the bosons are different from those in the strange metal phase and lead to modified gauge field fluctuations. As a result, the spin gap phase can be stabilized in the quantum critical (QC) and quantum disordered (QD) regime of the boson system. We also show that the regime of QD bosons with the paired fermions can be regarded as a strong coupling version of the recently proposed nodal liquid theory.

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The pseudogap behavior of underdoped cuprates in various physical properties has been a subject of intensive research recently [1]. The importance of the subject comes from the fact that understanding this behavior and relating it to the properties of the superconducting state may give us an important clue for the mechanism of the superconductivity. Among many different theories for the pseudogap, one of the earliest proposals was the mean field theory of the $t - J$ model which is an effective low energy theory of the Hubbard model in the limit of large on-site Coulomb repulsion [2].

In order to explain why this mean field theory is appealing, let us begin with the slave boson representation of the $t - J$ model. In the strong coupling limit, the double occupancy of the electrons at each site is prohibited, thus it is convenient to describe the Hilbert space of the electrons in terms of a neutral spin 1/2 fermion, $f_{i\alpha}^\dagger$, representing the singly occupied sites with a spin up or spin down electron, and a spinless charge e boson, b_i , keeping track of empty sites. As a result, the electron operator can be written as $c_{i\alpha}^\dagger = f_{i\alpha}^\dagger b_i$ with the constraint $\sum_\alpha f_{i\alpha}^\dagger f_{i\alpha} + b_i^\dagger b_i = 1$. The $t - J$ model can be written as

$$\begin{aligned} \mathcal{L} = & \sum_{i\alpha} f_{i\alpha}^\dagger (\partial_\tau - \mu) f_{i\alpha} + \sum_i b_i^\dagger (\partial_\tau - iA_0) b_i \\ & - t \sum_{\langle ij \rangle, \alpha} e^{-iA_{ij}} b_i b_j^\dagger f_{i\alpha}^\dagger f_{j\alpha} - J \sum_{\langle ij \rangle, \alpha\beta} f_{j\alpha}^\dagger f_{i\alpha} f_{i\beta}^\dagger f_{j\beta} \\ & - i \sum_i a_{0i} \left(\sum_\alpha f_{i\alpha}^\dagger f_{i\alpha} + b_i^\dagger b_i - 1 \right), \end{aligned} \quad (1)$$

where $A_{ij} = \int_i^j \mathbf{A} \cdot d\mathbf{l}$ and A_0 represent the external vector and scalar potentials. Here a_{0i} is the Lagrange multiplier enforcing the local constraint. Using the Hubbard-Stratonovich transformation, the action can be rewritten as

$$\mathcal{L} = \sum_{\langle ij \rangle} \left[J |Q_{ij}|^2 + \frac{|\Delta_{ij}|^2}{J} \right] + \mathcal{L}_F + \mathcal{L}_B,$$

$$\begin{aligned} \mathcal{L}_F = & \sum_{i\alpha} f_{i\alpha}^\dagger (\partial_\tau - \mu - ia_{0i}) f_{i\alpha} \\ & - J \sum_{\langle ij \rangle, \alpha} \left[|Q_{ji}| e^{-ia_{ij}} f_{i\alpha}^\dagger f_{j\alpha} + \text{H.c.} \right] \\ & + \sum_{\langle ij \rangle, \alpha\beta} \left[\Delta_{ij} \epsilon^{\alpha\beta} f_{j\beta} f_{i\alpha} + \text{H.c.} \right], \\ \mathcal{L}_B = & \sum_i b_i^\dagger (\partial_\tau - ia_{0i} - iA_0) b_i \\ & - t \sum_{\langle ij \rangle} \left[|Q_{ji}| e^{-ia_{ij}} e^{-iA_{ij}} b_i^\dagger b_j + \text{H.c.} \right], \end{aligned} \quad (2)$$

where $Q_{ij} = |Q_{ij}| e^{ia_{ij}}$. In the mean field theory, it was found that $|Q_{ij}| = Q = \text{const.}$ and $a_{ij} = 0$. It has been established that there are four different phases at the mean field level depending on the values of Δ_{ij} and $\langle b_i \rangle$ [2]. i) Superconducting phase: $\Delta_{ij} = \langle \epsilon^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger \rangle \neq 0$ and $\langle b_i \rangle \neq 0$. ii) Spin gap phase: $\Delta_{ij} \neq 0$ and $\langle b_i \rangle = 0$. iii) ‘Fermi liquid’ phase: $\Delta_{ij} = 0$ and $\langle b_i \rangle \neq 0$. iv) ‘Strange metal’ phase: $\Delta_{ij} = 0$ and $\langle b_i \rangle = 0$. As the electron is a combination of the fermion and boson, an excitation gap of the electron will be generated from the gap of the spin carrying fermions in the spin gap phase. Therefore, this theory already suggests the spin gap phase as a possible candidate for the pseudogap behavior of underdoped cuprates.

However, it was later found by Ubbens and Lee that the spin gap phase of the mean field theory is unstable against the fluctuations about the mean field state when the spin gap phase is approached from the strange metal phase [3]. One can see from Eq.2 that the fluctuation in the phase of Q_{ij} can be represented in terms of a $U(1)$ gauge field \mathbf{a} through $a_{ij} = \int_i^j \mathbf{a} \cdot d\mathbf{l}$. This is associated with the internal $U(1)$ symmetry in the slave-boson representation of the $t - J$ model. The previous study mentioned above would imply that the $U(1)$ slave-boson theory does not support the spin gap phase.

In order to make the later discussion more concrete, let us first reproduce the arguments of Ubbens and Lee [3].

Upon approaching the spin gap phase from the strange metal phase, one can evaluate the free energy cost for opening up the gap for the fermions in the following fashion. First we evaluate the contribution to the free energy from the gauge field fluctuations in the strange metal phase.

$$F_g = \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi} (2n_B(\omega) + 1) \arctan \left(\frac{\text{Im} D^{-1}}{\text{Re} D^{-1}} \right), \quad (3)$$

where $D(\mathbf{q}, \omega)$ is the gauge field propagator and given by $D^{-1}(\mathbf{q}, \omega) = \Pi_F^{jj}(\mathbf{q}, \omega) + \Pi_B^{jj}(\mathbf{q}, \omega)$. Here Π_F^{jj} and Π_B^{jj} are the fermion and boson current-current correlation functions respectively. In the strange metal phase, Π_F^{jj} and Π_B^{jj} were assumed to have the free fermion and boson forms: $\Pi_F^{jj} = -i\gamma\omega/q + \chi_F q^2$ and $\Pi_B^{jj} \approx \chi_B q^2$. Here $\gamma = 2n_e/k_F$, $\chi_F = 1/(12\pi m_F)$, and $\chi_B = (e^{T_{BE}/T} - 1)/(24\pi m_B)$, where n_e is the electron density, $m_F = 1/(2JQ)$, $m_B = 1/(2tQ)$, and $T_{BE} = 2\pi x/m_B$ (x is the doping concentration) is the mean field boson condensation temperature. As a result, D^{-1} is given by

$$D^{-1} \approx -i\gamma \frac{\omega}{q} + \chi q^2, \quad (4)$$

where $\chi = \chi_F + \chi_B$. Using Eq.3 and Eq.4, we get $F_g \propto T^{5/3}$. When the gap is opened, Δ can be introduced to cutoff the frequency integral and the effect is simply replacing T by Δ in F_g . Thus, the free energy cost for opening up the spin gap is proportional to $\Delta^{5/3}$. Since the mean field pairing energy gain is proportional to $-\Delta^2$, the free energy cost from the gauge field always dominates. Thus the spin gap phase cannot be stabilized. In their work, the bosons in the strange metal phase were assumed to behave rather classically. The transition from the strange metal phase to the spin gap phase occurs due to the pairing of the fermions while the bosons were assumed to be still classical.

In this paper, we suggest that the spin gap phase can be stabilized at low temperatures because the properties of the bosons are *different* from those in the high temperature strange metal phase. Motivated by the phase diagram of the cuprates, we suggest that at zero temperature there exists a quantum disordering phase transition of the bosons driven by vortices while the fermions remain paired across the transition (Fig. 1). The low density boson system ($x < x_c$) becomes an insulator due to the condensation of vortices. On the other hand, if $x > x_c$, the bosons are in the superfluid state while the vortices are in the insulating state. Recall that the electron resistivity is given by $\rho_e = \rho_F + \rho_B$, where ρ_F and ρ_B are the fermion and boson resistivities respectively. Since $\rho_F = 0$ and ρ_B diverges when $x < x_c$, the zero temperature spin gap phase is an insulator. We found that, near the QC point of the phase transition, the properties of the bosons are significantly modified. Using the

boson-vortex duality, we obtain the current-current correlation function, Π_B^{jj} , for the QD bosons. Due to the modified bosonic properties, the gauge field propagator has a different form at low temperatures, which leads to, according to Eq.3, the free energy cost for opening the spin gap smaller than the pairing energy gain, $-\Delta^2$. Thus the spin gap phase can be stabilized. In addition, we suggest that the recently proposed nodal liquid theory can be regarded as a weak coupling version of our theory if the constraints are ignored [4].

Let us begin with the superconducting state where the fermions are paired ($\Delta_{ij} \neq 0$) and the bosons are in the superfluid state ($\langle b_i \rangle \neq 0$). The phase diagram of the cuprates tells us that the superconducting state exists when the doping concentration is larger than a particular value x_c . The question is whether the normal state in the regime $x < x_c$ can be obtained by suppressing the boson superfluid while the fermions are still paired. In order to answer this question, we have to understand the nature of the quantum disordering phase transition of the bosons across x_c . We suggest that the latter is due to the condensation of vortices. The vortices carry the flux quantum hc/e because the bosons carry charge e . In order to describe the QD state of bosons, it is convenient to use the dual representation of the bosons [5]. Notice that the external electromagnetic fields only couple to the bosons. The continuum limit of \mathcal{L}_B in the presence of the external fields is given by

$$\mathcal{L}_B = b^\dagger (\partial_\tau - ia_0 - iA_0)b - \frac{1}{2m_B} b^\dagger (\nabla - i\mathbf{a} - i\mathbf{A})^2 b. \quad (5)$$

Following Ref. [5], let $b = \sqrt{\rho}\phi$ where ρ is positive definite, which corresponds to the boson density, and ϕ is a unimodular complex field satisfying $\phi^\dagger \phi = 1$. Then the action (Eq.5) becomes

$$\mathcal{L}_B = i\rho(\phi^\dagger \frac{\partial_\tau}{i} \phi - a_0 - A_0) + \frac{\rho}{2m_B} \left| \phi^\dagger \frac{\nabla}{i} \phi - \mathbf{a} - \mathbf{A} \right|^2 + \frac{1}{2m_B} |\nabla \sqrt{\rho}|^2. \quad (6)$$

Next, we decouple the second term by a Hubbard-Stratonovich transformation, which leads to

$$\mathcal{L}_B = iJ_\mu(\phi^\dagger \frac{\partial_\mu}{i} \phi - a_\mu - A_\mu) + \frac{m_B}{2\rho} |\mathbf{J}|^2 + \frac{1}{2m_B} |\nabla \sqrt{\rho}|^2, \quad (7)$$

where $J_\mu = (\rho, \mathbf{J})$ is the boson three-current and $A_\mu = (A_0, \mathbf{A})$. In order to isolate the vortices, we write ϕ as $\phi = \phi_v e^{i\theta}$, where θ is single valued and ϕ_v represents the vortices. Integration over θ gives the continuity constraint $\partial_\mu J_\mu = 0$. One can solve this constraint by introducing a new field M_λ through $J_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu M_\lambda \equiv (\partial \times M)_\mu$. It is also useful to introduce $\delta M_\mu = M_\mu - \bar{M}_\mu$

with $(\partial \times \bar{M})_0 = x$ and $(\partial \times \bar{M})_{1,2} = 0$, where x is the average boson density. Then the action in the long wavelength limit can be written as

$$\mathcal{L}_B = \frac{m_B}{2x} (|(\partial \times \delta M)_1|^2 + |(\partial \times \delta M)_2|^2) + i \delta M_\mu [J_\mu^v - (\partial \times a)_\mu - (\partial \times A)_\mu], \quad (8)$$

where $(\partial \times a)_\mu \equiv \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda$ and J_μ^v is the vortex three-current $J_\mu^v = \epsilon_{\mu\nu\lambda} \partial_\nu \phi_v^\dagger \frac{\partial}{\partial i} \phi_v \equiv (\partial \times \phi_v^\dagger \frac{\partial}{\partial i} \phi_v)_\mu$. When the vortices are condensed, the vortex transverse current-current correlation function in the long wavelength and low energy limit should have the form: $\langle J^v J^v \rangle = C \rho_v^s(T, x)$, where $\rho_v^s(T, x)$ is the superfluid density of the vortex condensate and C is a constant. Using this correlation function and integrating out M_μ degrees of freedom, the effective action for the transverse part of $a_\mu + A_\mu$ field becomes

$$\mathcal{L}_{B,\text{eff}} = \sum_{\mathbf{q}, \omega} \frac{q^2}{C \rho_v^s(T, x)} |(\mathbf{a} + \mathbf{A})_{\mathbf{q}, \omega}|^2. \quad (9)$$

Comparing with Eq.5, we can read off the boson current-current correlation function at finite temperatures:

$$\Pi_B^{jj} = \frac{q^2}{C \rho_v^s(T, x)}. \quad (10)$$

Since ρ_v^s becomes smaller and smaller as the critical point is approached, if we write down Π_B^{jj} as $\Pi_B^{jj} = \chi_B q^2$ with $\chi_B \propto 1/C \rho_v^s$, we have $\chi_F \ll \chi_B$ near the critical point. Then the gauge field propagator at finite temperatures can be written as

$$D^{-1} \approx -i\gamma\omega/q + \frac{q^2}{C \rho_v^s}. \quad (11)$$

Using the above propagator and Eq.3, we obtain

$$F_g \propto [\rho_v^s(T, x)]^{2/3} T^{5/3}. \quad (12)$$

Now some remarks on the temperature dependence of $\rho_v^s(T, x)$ are in order. Using dimensional analysis of hyperscaling, *i.e.* the free energy within a correlation volume $\xi^d \xi^z$ is non-singular in the vicinity of the quantum critical point, the scaling form of $\rho_v^s(T, x)$ in d -dimensions can be written as [6]

$$\rho_v^s(T, x) = \xi^{2-d-z} F(\xi^z T), \quad (13)$$

where $\xi \propto |x - x_c|^{-\nu}$ is the correlation length of the vortex superfluid and $F(x)$ is a scaling function. Here z is the dynamical critical exponent and ν is the correlation length exponent. If $\xi^z T \gg 1$, *i.e.* in the quantum critical regime, the vortex superfluid density must be independent of ξ and only depend on temperature. In two dimensions, this implies $F(x) \sim x$ for $x \gg 1$, leading to $\rho_v^s \propto T$. Substituting the latter into Eq.12, we find that the free energy cost for opening up the spin gap due to

the gauge field becomes $F_g \sim T^{7/3}$ in the QC regime. Similarly, in the quantum disordered regime, $\xi^z T \ll 1$, the scaling function $F(x) \rightarrow \text{const}$ for $x \ll 1$, ρ_v^s becomes temperature independent $\rho_v^s \propto \xi^{-z}$. In this case, Eq.12 becomes $F_g \propto \xi^{-2z/3} T^{5/3}$. When $T < \Delta$, one would use Δ as a low frequency cutoff instead of T . As long as $\Delta > B|x - x_c|^{z\nu}$, one can trade T and ξ^{-z} with Δ in the above expression to obtain the energy cost for opening up the gap $F_g \propto \Delta^{7/3}$. Thus the mean field fermion pairing energy gain, $-\Delta^2$, wins and the spin gap phase can be stabilized in both the QC and the QD regimes. We stress that this is a *qualitatively* very different behavior compared to the previous claim [3].

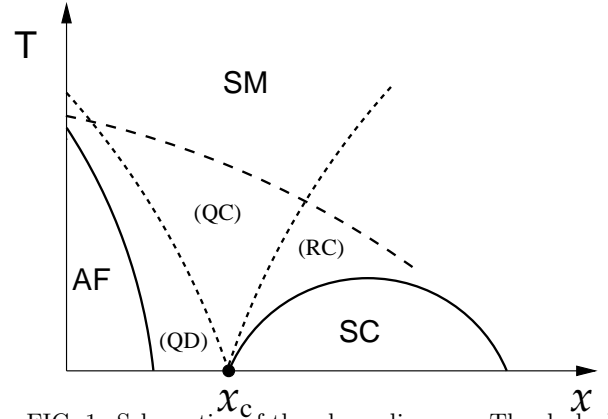


FIG. 1. Schematics of the phase diagram. The dashed lines indicate the crossover between QC and QD, QC and RC, and QC/RC and the strange metal (SM) regimes. The spin gap is stable in the QC and QD regimes.

At sufficiently high temperatures, the bosons would behave classically. In this regime, the argument of Ubbens and Lee can be applied [3]. Therefore, there should be a crossover from either QD or QC regime to the high temperature strange metal phase where the gauge field fluctuations destroy the spin gap phase. Fig. 1 shows a schematic phase diagram in the $T-x$ plane. Notice that there exists a crossover line between the strange metal phase and the spin gap phase stabilized in both the QD or QC regime of bosons. The dashed lines ($T = B|x - x_c|^{z\nu}$ line) represent the crossovers between the QD, QC, and renormalized classical (RC) regimes of bosons. Since the bosons in the RC regime behave essentially in a classical fashion, the spin gap phase is not stable there. Thus, the spin gap phase is restricted to the underdoped regime. The crossover from the QC regime to RC regime occurs when $T < B(x - x_c)^{z\nu}$ and $x > x_c$. Thus the transition from the strange metal phase to the superconducting phase is likely to occur in the RC regime (*i.e.*, T_c may be lower than $B(x - x_c)^{z\nu}$) so that it is characterized by a more or less classical transition of the bosons to the superfluid state. On the other hand, if $T < B(x_c - x)^{z\nu}$ and $x < x_c$, the crossover from the QC regime to the QD state of bosons occurs. Thus the antiferromagnetic phase is in the quantum disordered regime of the bosons. In the

low temperature limit, the relevant excitations in d-wave spin gap phase would be the neutral Dirac fermions. At lower doping concentrations, the $SU(2)$ fluctuations [7] may become important and these neutral Dirac fermions may be confined when x is sufficiently small and the antiferromagnetism occurs [7,8].

Now let us discuss the relation between the present theory and the nodal liquid theory proposed recently, where the quantum disordering of Cooper pairs was discussed [4]. Let us begin with the continuum limit of \mathcal{L}_F assuming that $\Delta_{ij} = \Delta$ is a uniform complex number and we will comment on the d-wave case later:

$$\mathcal{L}_F = f_\alpha^\dagger (\partial_\tau - \mu - ia_0) f_\alpha - \frac{1}{2m_F} f_\alpha^\dagger (\nabla - i\mathbf{a})^2 f_\alpha + \Delta \epsilon^{\alpha\beta} f_\beta f_\alpha + \Delta^* \epsilon^{\alpha\beta} f_\alpha^\dagger f_\beta^\dagger. \quad (14)$$

Let $\Delta = |\Delta|e^{i\phi}$. After doing the gauge transformation $f_\alpha \rightarrow f_\alpha e^{i\phi/2}$ and $b \rightarrow b e^{i\phi/2}$, we obtain the action in the absence of the external fields:

$$\mathcal{L}_F = f_\alpha^\dagger (\partial_\tau - \mu - i\tilde{a}_0) f_\alpha - \frac{1}{2m_F} f_\alpha^\dagger (\nabla - i\tilde{\mathbf{a}})^2 f_\alpha + |\Delta| (\epsilon^{\alpha\beta} f_\beta f_\alpha + \epsilon^{\alpha\beta} f_\alpha^\dagger f_\beta^\dagger), \quad (15)$$

$$\mathcal{L}_B = b^\dagger (\partial_\tau - i\tilde{a}_0) b - \frac{1}{2m_B} b^\dagger (\nabla - i\tilde{\mathbf{a}})^2 b,$$

where $\tilde{a}_\mu = a_\mu + (\partial_\mu \phi)/2$. This amounts to fixing the gauge for a_μ . In the superconducting state, the order parameter in the strong coupling limit can be written as $\Delta^e = \langle \epsilon^{\alpha\beta} c_\alpha^\dagger c_\beta^\dagger \rangle \approx \langle \epsilon^{\alpha\beta} f_\alpha^\dagger f_\beta^\dagger \rangle \langle bb \rangle$. In the gauge choice we have taken, $\langle \epsilon^{\alpha\beta} f_\alpha^\dagger f_\beta^\dagger \rangle$ is always real. Therefore, in the superconducting state, we get $\Delta^e \approx |\Delta| \langle bb \rangle \approx |\Delta| b_0^2 e^{i\theta}$. Here $b = b_0 e^{i\theta/2}$ is taken. Notice that the phase of Δ is dictated by the phase of the bosons and $|\Delta^e| = |\Delta| b_0^2$. In the superconducting state, we substitute b by $b_0 e^{i\theta/2}$ in the action. Integrating out \tilde{a}_0, \mathbf{a} , we obtain

$$\mathcal{L} = \tilde{\mathcal{L}}_F + \mathcal{L}_\theta + \mathcal{L}_{F,\theta},$$

$$\tilde{\mathcal{L}}_F = f_\alpha^\dagger (\partial_\tau - \mu - \frac{1}{2m_F} \nabla^2) f_\alpha + |\Delta| (\epsilon^{\alpha\beta} f_\beta f_\alpha + \epsilon^{\alpha\beta} f_\alpha^\dagger f_\beta^\dagger) + \int d^3 r' J_\mu^F(r) (\Pi^F)_{\mu\nu}^{-1}(r-r') J_\nu^F(r'),$$

$$\mathcal{L}_\theta = \kappa_\mu (\partial_\mu \theta)^2,$$

$$\mathcal{L}_{F,\theta} = g_\mu J_\mu^F (\partial_\mu \theta), \quad (16)$$

where $J_\mu^F = (\rho^F, \mathbf{J}^F)$ with $\rho^F = f_\alpha^\dagger f_\alpha$ and $\mathbf{J}^F = \frac{1}{2im_F} (f_\alpha^\dagger \nabla f_\alpha - \text{H.c.})$. $\Pi_{\mu\nu}^F = \langle J_\mu^F J_\nu^F \rangle$ is the fermion three-current correlation function. Here $g_0 = 1/2$ and $g_{1,2} = (\frac{x}{m_B}) / [\frac{x}{m_B} + \frac{(1-x)}{m_F}]$. Also $\kappa_0 = N(0)^{-1} = \frac{2\pi}{m_F}$ and $\kappa_{1,2} = \frac{1}{2} (\frac{x}{m_B}) g_{1,2}$. The continuum action derived above can be also obtained directly from the lattice model given by Eq.1 [9].

If Δ_{ij} has the d-wave symmetry, the relevant degrees of freedom in the low energy limit are the excitations near

the nodes. These excitations have the Dirac spectra and can be represented by $d_a = (d_{a\uparrow}, d_{a\downarrow}^\dagger)$, where $a = 1, 2$ represent two pairs of nodes. Following Ref. [4], the fermion part of the action becomes

$$\mathcal{L}_d = d_1^\dagger (\partial_\tau - v_F \tau^z i \partial_x - v_\Delta \tau^x i \partial_y) d_1 + d_2^\dagger (\partial_\tau - v_\Delta \tau^z i \partial_y - v_F \tau^x i \partial_x) d_2, \quad (17)$$

where $\tau^{x,y,z}$ are the Pauli matrices and $v_\Delta = \Delta/\sqrt{2}$. The coupling between the fermion and the phase fields is also changed to $\mathcal{L}_{d,\theta} = \tilde{g}_\mu J_\mu^d (\partial_\mu \theta)$, where J_μ^d is the current of the Dirac fermion d . In this case, the corresponding action in Eq.16 looks similar to that of the nodal liquid theory if the additional terms coming from the constraints are ignored. Thus we conclude that our theory can be also interpreted as the strong coupling version of the nodal liquid theory. However, due to the presence of the constraints, the structure of the theory is not exactly the same. Notice also that the presence of the hc/e vortices as well as the $hc/2e$ vortices was pointed out by Sachdev [10] as a consequence of the constraint imposed by the gauge field. We believe that the presence of the hc/e vortices suggested by the nodal liquid theory comes out naturally in the present strong coupling theory. It was also suggested that the vortex condensate supports charge e solitons (dual vortex of the vortex condensate), *holons* [4] in the dual picture. Using the boson-vortex duality, one can see that the slave-bosons in the boson picture may be the natural candidates for the solitonic excitations in the dual representation.

In summary, using the $U(1)$ slave-boson representation of the $t - J$ model, we obtained the spin gap phase by quantum disordering the slave-boson superfluid of the superconducting phase while the spin-carrying neutral fermions are paired. We found that the spin gap phase can be stabilized in the QC and QD regime of the bosons against the fluctuations about the mean field state. We also showed that the spin gap phase at zero temperature obtained in this fashion can be regarded as a strong coupling version of the nodal liquid phase.

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